

Last name:

First name:

BU or BG:

Do not use any software or calculator. All work is to be individual and without consulting book, the web, or any other resources.

Problem 1 (5 points): Using any of the techniques learned in class, draw, to the best of your ability, the phase portrait of

$$\begin{aligned}x' &= x + y^2 \\ y' &= y\end{aligned}$$

Problem 2 (10 points): There is a bifurcation for a value of $a = a_c$ in the system below. Find such value. Draw the phase portrait for a value of $a < a_c$, $a = a_c$ and for a value of $a > a_c$. Describe the bifurcation.

$$\begin{aligned}x' &= x^2 + y \\ y' &= x - y + a\end{aligned}$$

Problem 3 (10 points): The system below is in polar coordinates. Draw the phase portrait of the system below, for values of a smaller, equal and bigger to all the values where a bifurcation occurs.

$$\begin{aligned}r' &= r - r^2 \\ \theta' &= \sin^2(\theta) + a\end{aligned}$$